MARKSCHEME

May 2010

MATHEMATICS
DISCRETE MATHEMATICS

Higher Level

Paper 3

Samples to team leaders	June 10 2010
Everything (marks, scripts etc) to IB Cardiff	June 17 2010

10 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

  $$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

  Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Candidates should be penalized ***once only IN THE PAPER*** *for an accuracy error* ***(AP)****. Award the marks as usual then write* ***(AP)*** *against the answer. On the* ***front*** *cover write –1****(AP)****. Deduct 1 mark from the total for the paper, not the question.*

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

1. (a) (i) $4^8 = 65536 \equiv 7 \pmod 9$ ***A1***

not valid because 9 is not a prime number ***R1***

Note: The ***R1*** is independent of the ***A1***.

(ii) using Fermat's little theorem ***M1***

$5^6 \equiv 1 \pmod 7$ ***A1***

therefore

$(5^6)^{10} = 5^{60} \equiv 1 \pmod 7$ ***A1***

also, $5^4 = 625$ ***M1***

$\equiv 2 \pmod 7$ ***A1***

therefore

$5^{64} \equiv 1 \times 2 \equiv 2 \pmod 7$ (so $n = 2$) ***A1***

Note: Accept alternative solutions not using Fermat.

[8 marks]

(b) **EITHER**

solutions to $x \equiv 3 \pmod 4$ are

$3, 7, 11, 15, 19, 23, 27, \ldots$ ***A1***

solutions to $3x \equiv 2 \pmod 5$ are

$4, 9, 14, 19 \ldots$ ***(M1)A1***

so a solution is $x = 19$ ***A1***

using the Chinese remainder theorem (or otherwise) ***(M1)***

the general solution is $x = 19 + 20n$ $(n \in \mathbb{Z})$ ***A1***

$\left(\text{accept } 19 \pmod{20}\right)$

OR

$x = 3 + 4t \Rightarrow 9 + 12t \equiv 2 \pmod 5$ ***M1A1***

$\Rightarrow 2t \equiv 3 \pmod 5$ ***A1***

$\Rightarrow 6t \equiv 9 \pmod 5$

$\Rightarrow t \equiv 4 \pmod 5$ ***A1***

so $t = 4 + 5n$ and $x = 19 + 20n$ $(n \in \mathbb{Z})$ ***M1A1***

$\left(\text{accept } 19 \pmod{20}\right)$

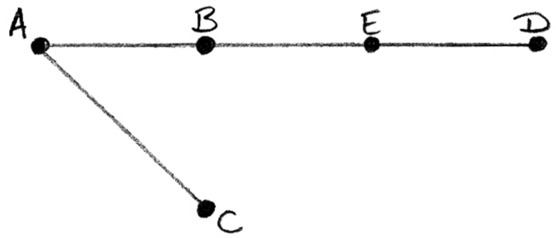
Note: Also accept solutions done by formula.

[6 marks]

Total [14 marks]

2. (a) (i) the edges are joined in the order

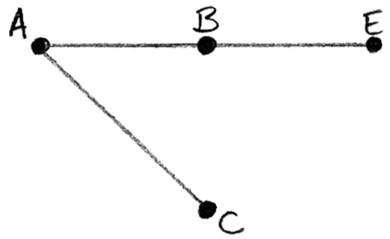
AC
BE
AB
ED *A2*



A1

Note: Final *A1* independent of the previous *A2*.

(ii)



A1

the weight of this spanning tree is 33 *A1*

to find a lower bound for the travelling salesman problem, we add to that the two smallest weights of edges to D, *i.e.* $15+16$, giving 64 *M1A1*

[7 marks]

(b) an upper bound is the weight of any Hamiltonian cycle, *e.g.* ABCDEA has weight 75 so 80 is certainly an upper bound *M1A1*

[2 marks]

Total [9 marks]

3. (a) let $N = a_n a_{n-1} ... a_1 a_0 = a_n \times 9^n + a_{n-1} \times 9^{n-1} + ... + a_1 \times 9 + a_0$ ***M1A1***
all terms except the last are divisible by 3 and so therefore is their sum ***R1***
it follows that N is divisible by 3 if a_0 is divisible by 3 ***AG***

[3 marks]

(b) **EITHER**

consider N in the form

$$N = a_n \times (9^n - 1) + a_{n-1} \times (9^{n-1} - 1) + ... + a_1 (9 - 1) + \sum_{i=0}^{n} a_i$$ ***M1A1***

all terms except the last are even so therefore is their sum ***R1***

it follows that N is even if $\sum_{i=0}^{n} a_i$ is even ***AG***

OR

working modulo 2, $9^k \equiv 1 (\text{mod } 2)$ ***M1A1***

hence $N = a_n a_{n-1} ... a_1 a_0 = a_n \times 9^n + a_{n-1} \times 9^{n-1} + ... + a_1 \times 9 + a_0 \equiv \sum_{i=0}^{n} a_i (\text{mod } 2)$ ***R1***

it follows that N is even if $\sum_{i=0}^{n} a_i$ is even ***AG***

[3 marks]

(c) the number is divisible by 3 because the least significant digit is 3 ***R1***
it is divisible by 2 because the sum of the digits is 44 which is even ***R1***
dividing the number by 2 gives $(232430286)_9$ ***M1A1***
which is even because the sum of the digits is 30 which is even ***R1***
N is therefore divisible by a further 2 and is therefore divisible by 12 ***R1***

Note: Accept alternative valid solutions.

[6 marks]

Total [12 marks]

4. (a) start with a graph consisting of just a single vertex *M1*
for this graph, $v=1$, $f=1$ and $e=0$, the relation is satisfied *A1*

Note: Allow solutions that begin with 2 vertices and 1 edge.

to extend the graph you either join an existing vertex to another existing vertex which increases e by 1 and f by 1 so that $v+f-e$ remains equal to 2 *M1A1*

or add a new vertex and a corresponding edge which increases e by 1 and v by 1 so that $v+f-e$ remains equal to 2 *M1A1*

therefore, however we build up the graph, the relation remains valid *R1*

[7 marks]

(b) since every face is bounded by at least 3 edges, the result follows by counting up the edges around each face *R1*
the factor 2 follows from the fact that every edge bounds (at most) 2 faces *R1*
hence $3f \le 2e$ *AG*

from the Euler relation, $3f = 6+3e-3v$ *M1*

substitute in the inequality, $6+3e-3v \le 2e$ *A1*
hence $e \le 3v-6$ *AG*

[4 marks]

(c) let G have e edges *M1*

since G and G' have a total of $\binom{12}{2}=66$ edges *A1*

it follows that G' has $66-e$ edges *A1*

for planarity we require
$e \le 3\times12-6=30$ *M1A1*
and $66-e \le 30 \Rightarrow e \ge 36$ *A1*

these two inequalities cannot both be met indicating that both graphs cannot be planar *R1*

[7 marks]

Total [18 marks]

5. **EITHER**

we work modulo 3 throughout
the values of a, b, c, d can only be $0, 1, 2$ ***R2***

since there are 4 variables but only 3 possible values, at least 2 of the variables must be equal $(\bmod 3)$ ***R2***

therefore at least 1 of the differences must be $0(\bmod 3)$ ***R2***

the product is therefore $0(\bmod 3)$ ***R1AG***

OR

we attempt to find values for the differences which do not give $0(\bmod 3)$ for the product

we work modulo 3 throughout
we note first that none of the differences can be zero ***R1***
$a-b$ can therefore only be 1 or 2 ***R1***
suppose it is 1, then $b-c$ can only be 1
since if it is 2, $(a-b)+(b-c)\equiv 3\equiv 0(\bmod 3)$ ***R1***

$c-d$ cannot now be 1 because if it is
$(a-b)+(b-c)+(c-d)=a-d\equiv 3\equiv 0(\bmod 3)$ ***R1***

$c-d$ cannot now be 2 because if it is
$(b-c)+(c-d)=b-d\equiv 3\equiv 0(\bmod 3)$ ***R1***

we cannot therefore find values of c and d to give the required result ***R1***
a similar argument holds if we suppose $a-b$ is 2, in which case $b-c$ must be 2 and we cannot find a value of $c-d$ ***R1***
the product is therefore $0(\bmod 3)$ ***AG***

[7 marks]